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SURFACE CHARGE FLOW IN 2-DIMENSIONAL POSITION SENSITIVE SILICON DETECTORS

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Abstract

A model has been constructed to study the electrode current characteristics of the resistive sheet type of 2-dimensional position sensitive silicon detector (2D-PSD) including those with a low resistance ring around the central high resistance area. The 2D-PSD is modeled as a regular mesh of discrete resistors connected at their nodes via capacitors to a common plane; a solution of the individual node voltages for short time steps allows for a determination of the electrode currents.

Position reconstruction non-linearities and energy signal ballistic deficit have been investigated for various values of surface, ring and electrode termination resistances by integrating the current pulses from a simulated particle hit.

It has been found that for a 2D-PSD with capacitance, C, and a ring resistance of $\frac{1}{10}^{th}$ of surface resistance, $R_{\Box}$, the position signals should be allowed to develop for $\approx R_{\Box}C/5$ seconds in order to obtain 1% non-linearity (r.m.s.) in the position pattern, and the energy signal for longer than $2R_{\Box}C/5$ seconds for less than 1% ballistic deficit.

A method is described that allows the model results to be used to correct position non-linearities that arise during experimental measurements.

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I. INTRODUCTION

A new Multi-Element Gas-Hybrid Array (MEGHA) [1] has recently been commissioned for the study of exotic cluster structures in nuclei. These studies are carried out using break-up reactions [2]. The nucleus under study is excited above the particle decay threshold - breaking up into two or more fragments. From a knowledge of the energy and angle of emission of these fragments properties such as the excitation energy spectrum of the excited nucleus can be determined [3]. In MEGHA, for low multiplicity studies, measurement of the energy and angle of emission are carried out using large area ($50 \times 50\ mm$) two-dimensional position sensitive silicon detectors (2D-PSD's). The operation and construction of these detectors has been reported by several authors [4–8].

Early 2D-PSD's used a single square resistive silicon sheet as the front electrode. Four signals, taken from the corners, were used to determine the position of an incident ion. This type of detector suffered from severe position non-linearities and required lengthy and complex corrections [8] to be useful for experimental work.

Doke *et al* [4] described a resistive sheet 2D-PSD, based on an idea by Gear [9], with approximately linear response. The original resistive sheet suggested by Gear had a four pointed star geometry shown schematically in figure 1, with the four corners linked via lower resistance curved strips of radius a. The position of incidence of a charged particle or photon could be determined *exactly*, *only* if the condition,

$$\frac{R_\square}{LR_L} = \frac{a}{L} \tag{1.1}$$

was satisfied, where L is the length of the low resistance strip, $R_\square$ is the surface resistance, and R_L the resistance per unit length of the resistive strip. Under these conditions the position of the incident ion is given by,

$$X = \left\{ \frac{(I_c + I_b) - (I_a + I_d)}{(I_a + I_b + I_c + I_d)} \right\} \tag{1.2}$$

$$Y = \left\{ \frac{(I_c + I_d) - (I_a + I_b)}{(I_a + I_b + I_c + I_d)} \right\} \tag{1.3}$$

where $I_{a \to d}$ are the total integrated electrode currents.

Detectors of this type are impractical for use in a compact detector array due to the complex shape and large non-sensitive surface area. To overcome this Doke investigated the position response of 2D-PSD's with $a \to \infty$, that is, for a square resistive sheet. The condition given in equation 1.1 can only be satisfied for these detectors if the thin resistive strips have zero resistance. It was shown that this led to poor position resolution [4,5]. However, Doke found that with values of $R_\square/LR_L$ as low as $10 \to 15$ the maximum distortion in the position pattern was less than 2% at the corners. Values of $R_\square/LR_L$ much greater than this resulted in non-linearities arising from inhomogeneities in the Silicon resistivity. However, the method Doke used to generate the position patterns approximated infinite charge collection time, and hence neglected the electrode current characteristics as analysed by finite-time pulse shaping networks.

The detectors used in MEGHA have values of $R_\square/LR_L \sim 10$. Measurements with these detectors show that with finite charge collection times the position linearity can change after long term irradiation and that the energy signal, taken from the rear face of the 2D-PSD, is position dependent, leading to the so-called ballistic deficit [10].

In an attempt to understand the charge collection processes in resistive sheet 2D-PSD's Hasebe *et al* [5] developed a model in which the resistance of the low resistance ring was constrained to be zero, that is $R_L = 0$. Charge arriving at the detector edges was instantly delivered to the electrodes - neglecting the characteristics of charge flow in the detector perimeter. Hasebe found an analytical solution to the charge diffusion equation for an RC-transmission 'line' described by the model, where C is the 2D-PSD capacitance. He discovered that after the charge was collected and single integration - differentiation pulse shaping applied, the ballistic deficit in the energy signal was less than 1% for a pulse shaping time constant of $R_\square C/3$. However, due to the approximation that $R_L = 0$ the model could not provide any information on position non-linearities.

Yanagimachi *et al* [6], using a 2D-PSD with $R_\square/LR_L \approx 13$ reported position non-linearities of 0.75% r.m.s. for $2R_\square C/3$ shaping times and 0.47% r.m.s. for $4R_\square C/3$ shaping times, but was unable to comment on the value of the ballistic deficit in the energy signal. He noted that as the pulse shaping time constant, τ_f, was increased the improvement in linearity was offset by a degradation in the position resolution.

In this paper a more realistic model is described which accurately simulates the characteristics of both the position and energy signals of 2D-PSD's. The response of the 2D-PSD is determined as a function of incident particle position for a variety of values of $R_\square/LR_L$. The effects of position electrode termination resistors and finite signal development times are also considered. This allows for a determination of the optimum detector characteristics to achieve a minimum of ballistic deficit and position non-linearity. This paper also describes a method in which the model data is used to correct for the distortions arising in experimental data using 2D-PSD's.

II. THE MODEL

A. Formulation

Figure 2 illustrates how the main resistive sheet and the low resistance ring are modeled. A square, capacitive anode with resistance, $R_\square$ per unit square, capacitance, C, and sides of length, L, is divided into M slices of equal width, $\Delta x = \Delta y$, in the horizontal and vertical directions. The resistance, r_s, joining the centres of adjacent segments is,

$$r_s = R_\square \tag{2.1}$$

and the capacitance of each segment is,

$$c_s = \frac{C}{M^2} \tag{2.2}$$

The product $r_s c_s M^2$ is equal to the characteristic detector time constant $R_\square C$.

For an anode with a perimeter ring of resistance, R_L per unit length, the resistance joining adjacent segments along the edge of the anode is,

$$r_l = \frac{L}{M} R_L \tag{2.3}$$

The ratio of the surface resistance, $R_\Box$, to the total line resistance linking two adjacent electrodes, LR_L, shown by Doke to be the parameter governing position linearity, is given by,

$$\frac{R_\Box}{LR_L} = \frac{r_s}{Mr_l} \tag{2.4}$$

The 2D-PSD is now modeled as a regular mesh of discrete resistors connected at their nodes via capacitors to a common plane - representing the rear ohmic contact side of the 2D-PSD. The resistance of this plane is very much less than the sheet resistance of the junction side and has therefore been approximated to zero. The nodal labeling, $(0 < m < M-1, 0 < n < M-1)$, is defined in figure 2.

The initial condition, used to simulate an incident particle, is approximated by assuming that the time-scale for charge generated during an ionizing event to migrate to the front and rear surfaces of the detector junction is much less than the typical diffusion times for charge flow across the detector face; correspondingly, in the model the charge at a specific nodal 'hit co-ordinate', labeled (m', n'), is set to unity at time $t = 0$.

B. Charge flow calculations

For time $t > 0$ the charge deposited on the nodal mesh begins to migrate to the four earth potentials at the corners through the resistor network. The charge, $q_{m,n}$, at a general internal node ($m, n \neq 0$ or M) is calculated as follows, referring to the nodal diagram illustrated in figure 3*(a)*; the currents $i_{m\to m+1}, i_{m-1\to m}, i_{n\to n+1}$ and $i_{n-1\to n}$ flow through the resistors surrounding the node (m, n) due to the potential difference between adjacent nodes, $\Delta q/c$,

$$\begin{aligned}
\dot{i}_{m\to m+1} &= \frac{(q_{m+1,n} - q_{m,n})}{r_s c_s} = \frac{M^2(q_{m+1,n} - q_{m,n})}{R_\Box C} \\
\dot{i}_{m-1\to m} &= \frac{(q_{m,n} - q_{m-1,n})}{r_s c_s} = \frac{M^2(q_{m,n} - q_{m-1,n})}{R_\Box C} \\
\dot{i}_{n\to n+1} &= \frac{(q_{m,n+1} - q_{m,n})}{r_s c_s} = \frac{M^2(q_{m,n+1} - q_{m,n})}{R_\Box C} \\
\dot{i}_{n-1\to n} &= \frac{(q_{m,n} - q_{m,n-1})}{r_s c_s} = \frac{M^2(q_{m,n} - q_{m,n-1})}{R_\Box C}
\end{aligned} \tag{2.5}$$

The current flow from the capacitor at an internal node is equal to the rate of change of nodal charge,

$$i_{m,n} = \left(\frac{\partial}{\partial t} q_{m,n}\right) \tag{2.6}$$

Balancing current flow into and out-of the node and rearranging gives,

$$\begin{aligned}\frac{\partial}{\partial t} q_{m,n} &= \frac{1}{r_s c_s}\left(q_{m-1,n} - 2q_{m,n} + q_{m+1,n} + q_{m,n-1} - 2q_{m,n} + q_{m,n+1}\right) \\ &= \frac{L^2}{R_\Box C}\left(\frac{q_{m-1,n} - 2q_{m,n} + q_{m+1,n}}{(\Delta x)^2}\right) + \frac{L^2}{R_\Box C}\left(\frac{q_{m,n-1} - 2q_{m,n} + q_{m,n+1}}{(\Delta y)^2}\right)\end{aligned} \tag{2.7}$$

This is the finite difference approximation to the 2-dimensional charge diffusion equation,

$$\frac{\partial}{\partial t} q(x,y) = \frac{L^2}{R_\Box C}\left(\frac{\partial^2}{\partial x^2} q(x,y) + \frac{\partial^2}{\partial y^2} q(x,y)\right) \tag{2.8}$$

Equation 2.7 may also be deduced [11] by considering a Taylor series expansion of $q(x,y)$ about the points $(y_m + \Delta y/2)$, $(x_m + \Delta x/2)$, y_m and x_m, where $\Delta y = y_m - y_{m-1}$ and $\Delta x = x_m - x_{m-1}$.

Current flow at the mesh edges and corners may be calculated using the appropriately truncated nodal diagrams, illustrated in figures 3*(b)* and *(c)*, ignoring at present the effects of the low resistance ring and termination resistors. The current balance equation at a general perimeter node $(m,n) = (0,n), (M,n), (m,0)$ or (m,M) for example node $(0,n)$, is given by,

$$\begin{aligned}\frac{\partial}{\partial t} q_{0,n} &= \frac{1}{r_s c_s}\left(-2q_{0,n} + 2q_{1,n} + q_{0,n-1} - 2q_{0,n} + q_{0,n+1}\right) \\ &= \frac{L^2}{R_\Box C}\left(\frac{-2q_{0,n} + 2q_{1,n}}{(\Delta x)^2}\right) + \frac{L^2}{R_\Box C}\left(\frac{q_{0,n-1} - 2q_{0,n} + q_{0,n+1}}{(\Delta y)^2}\right)\end{aligned} \tag{2.9}$$

and at a corner node $(m,n) = (0,0), (0,M), (M,0)$ or (M,M), for example node $(0,0)$, where the four corners without termination resistors are held at zero potential, by,

$$\begin{aligned}\frac{\partial}{\partial t} q_{0,0} &= \frac{1}{r_s c_s}\left(2q_{1,0} + 2q_{0,1}\right) = 0 \\ &= \frac{L^2}{R_\Box C}\left(\frac{2q_{1,0}}{(\Delta x)^2}\right) + \frac{L^2}{R_\Box C}\left(\frac{2q_{0,1}}{(\Delta y)^2}\right) = 0\end{aligned} \tag{2.10}$$

Calculation of the exact solution of the M^2 differential equations, resulting from a simultaneous consideration of all nodes, quickly becomes complex and lengthy for large M. Therefore, a further application of the finite difference method has been made to approximate the temporal development of the nodal charges.

The solution for $q_{m,n}$ at time $t = (t + \Delta t)$ is given by the Euler approximation,

$$q_{m,n}^{(t+\Delta t)} = q_{m,n}^{(t)} + \left(\frac{\partial}{\partial t} q_{m,n}^{(t)}\right) \Delta t \tag{2.11}$$

At time $t = 0$ all the charge is concentrated at the nodal hit co-ordinate (m', n'),

$$q_{m,n}^{(0)} = \begin{cases} 1 \text{ , when } (m,n) = (m',n') \\ 0 \text{ , elsewhere} \end{cases} \tag{2.12}$$

By substituting equations 2.7, and the general forms of equations 2.9 and 2.10 into equation 2.11 and writing in matrix form, an expression is derived which is an approximation of the temporal development of the nodal charges for a 2D-PSD with a uniform sheet resistance,

$$\mathbf{q}^{(t+\Delta t)} = \mathbf{k}.\mathbf{q}^{(t)} + \{\mathbf{k}.\mathbf{q}^{(t)\mathbf{T}}\}^{\mathbf{T}} + \mathbf{q}^{(t)} \tag{2.13}$$

where $\mathbf{k}$ and $\mathbf{q}$ are the $M \times M$ matrices,

$$\mathbf{k} = \begin{bmatrix} -2p & 2p & 0 & 0 & \cdots \\ p & -2p & p & 0 & \\ & & \ddots & & \\ & 0 & p & -2p & p \\ \cdots & 0 & 0 & 2p & -2p \end{bmatrix} \text{ and, } \mathbf{q} = \begin{bmatrix} q_{0,0} & q_{0,1} & \cdots & q_{0,M-1} \\ q_{1,0} & q_{1,1} & \cdots & q_{1,M-1} \\ & & \ddots & \\ q_{M-2,0} & \cdots & q_{M-2,M-2} & q_{M-2,M-1} \\ q_{M-1,0} & \cdots & q_{M-1,M-2} & q_{M-1,M-1} \end{bmatrix} \tag{2.14}$$

p is defined as the time-step $(\Delta t / R_{\square} C)$ and $\mathbf{T}$ is the matrix transpose.

Figures 4*(a)*, *(b)* and *(c)* show the calculated charge surface $\mathbf{q}^{(t)}$ at times *(a)* $t = 0$; *(b)* $t = 0.01 R_{\square} C$ and *(c)* $t = R_{\square} C$ for a mesh with $R_{\square}/LR_L = 1$ and $M = 11$ for the initial condition $q_{3,3}^{(t=0)} = 1$. When $t = 0$, equation 2.12 demands that a unit of charge exists at $(m'n') = (3,3)$. For $t > 0$ the charge begins to migrate towards the four virtual earths at the corner electrodes, eventually for $t \gg 0$ little of the charge is left on the mesh and the charge profile is symmetric about the mesh centre.

The low resistance ring can be modeled by introducing a further $M \times M$ matrix, $\mathbf{s}$, which increases the rate of flow of charge between nodes along the mesh perimeter only, by the factor $R_\Box / LR_L$,

$$\mathbf{s} = \begin{bmatrix} R_\Box/LR_L & 0 & 0 & 0 & \dots \\ 0 & 1 & 0 & 0 & \\ & & \ddots & & \\ & 0 & 0 & 1 & 0 \\ \dots & 0 & 0 & 0 & R_\Box/LR_L \end{bmatrix} \tag{2.15}$$

Now,

$$\mathbf{q}^{(t+\Delta t)} = \mathbf{k}.(\mathbf{q}^{(t)}.\mathbf{s}) + \{\mathbf{k}.(\mathbf{q}^{(t)\mathbf{T}}.\mathbf{s})\}^{\mathbf{T}} + \mathbf{q}^{(t)} \tag{2.16}$$

The charge profile for the case where $R_\Box / LR_L = 10$ is plotted in figure 5*(b)* at time $t = 0.02 R_\Box C$ for the nodal hit co-ordinate $(3, 3)$. There is very much less charge existing along the mesh perimeter compared to figure 5*(a)* for a uniform resistance mesh. The low resistance ring is enhancing charge collection along the detector edges as expected.

In experimental situations, termination resistors, often called offset resistors, are attached in series between the position electrodes and the input of charge sensitive preamplifiers, as shown in figure 6. The addition of these resistors allows for a minimum of resistive charge division to occur for events near the detector corners, so that the charge pulse in the furthest electrode rises above detection threshold for the lowest energy events. The value of the termination resistance, R_T, is set in relation to the value of sheet resistance $R_\Box$; generally, in experimental measurements the value of the termination resistance is set at $\approx \frac{1}{10}$ to $\frac{1}{2}$ of the sheet resistance.

In this model the effects of the termination resistors must be considered separately since they do not lie within the nodal framework. The potential generated at the four corner nodes is now non-zero, and equation 2.10 is modified by both a potential divider term and the low resistance ring factor, to give at the corner node $(0, 0)$ for example,

$$\frac{\partial}{\partial t} q_{0,0} = \left\{ \frac{L^2}{R_\Box C} \left(\frac{-2q_{0,0}\left(\frac{R_\Box + R_T}{R_T}\right) + 2q_{1,0}}{(\Delta x)^2} \right) + \frac{L^2}{R_\Box C} \left(\frac{-2q_{0,0}\left(\frac{R_\Box + R_T}{R_T}\right) + 2q_{0,1}}{(\Delta y)^2} \right) \right\} \frac{R_\Box}{LR_L} \tag{2.17}$$

The effect of the termination resistors is clearly seen in terms of the non-zero charge supported at the corner node in figure 5*(c)*, where $R_T = R_\Box/10$ and $R_\Box/LR_L$ =10.

The electrode currents can now be evaluated by determining the potential difference across the termination resistors. Figure 6 shows a schematic of the 2D-PSD; the currents, $i_{a\to d}$, flow through the four termination resistors, R_T , to the virtual earths of charge-sensitive preamplifiers (only one is drawn) due to the potential developed at the four corner nodes. These electrode currents,

$$\begin{aligned} i_{a;(m',n');t} &= \frac{4M^2 q_{0,0}}{CR_T} \\ i_{b;(m',n');t} &= \frac{4M^2 q_{M-1,0}}{CR_T} \\ i_{c;(m',n');t} &= \frac{4M^2 q_{0,M-1}}{CR_T} \\ i_{d;(m',n');t} &= \frac{4M^2 q_{M-1,M-1}}{CR_T} \end{aligned} \tag{2.18}$$

are dependent on nodal hit co-ordinate (m', n'). Once integrated these electrode currents can be used in equations 1.2 and 1.3 to measure the position of nodal charge origin or hit co-ordinate. The electrode labeling, $a \to d$, is defined in figure 6 and will be referred to throughout the text.

III. RESULTS

The model has been investigated principally for two mesh types;

- *mesh-I* simulates a 2D-PSD with uniform sheet resistance, ie. $R_\Box/LR_L = 1$, and
- *mesh-II* simulates a typical low resistance ring type 2D-PSD, with $R_\Box/LR_L = 10$

The value of M has been set at 11 to limit calculation time. An analysis of the effects of varying the mesh size has shown that for $M > 11$ the model output (for example the value of $q_{0,0}$) changes by less than 0.25% compared to the output for $M = 11$, see figure 7*(a)* (this dependency is mirrored in a comparison of all other model output variables), whilst the computation time increases approximately proportional to M^4, see figure 7*(b)*, due to the shortening of the time-step, $p = \Delta t / M^2 r_s c_s$, and the increased number of nodes, M^2.

A. Surface and electrode current flow

Figure 8 shows a plot of the calculated current flow in the four termination resistors of *mesh-II* as a function of time (in units of $R_\square C$). The position of charge origin is indicated schematically in the top right hand corner of the figure, representing the initial condition $q_{2,3}^{(t=0)} = 1$. The value of R_T is set at $R_\square/10$. The time to the peak pulse height increases for the electrodes further from the nodal hit co-ordinate, agreeing with the simple idea that it takes time for charge to diffuse across the detector. This position dependence of the pulse rise-times is the origin of the ballistic deficit.

Charge sensitive preamplifiers are used in experiments to convert the electrode current pulses into measurable voltage signals, see figure 6. The voltage output of the preamplifier is given by,

$$V(t) = \frac{\int_0^t i \; dt}{C_{PA}} \exp\left[-\left(\frac{t}{R_{PA} C_{PA}}\right)\right] \tag{3.1}$$

where the preamplifier feedback loop contains the resistor, R_{PA}, and capacitor, C_{PA}, in parallel. The product $R_{PA} C_{PA}$ is the capacitive discharge time constant of the preamplifier and is set much larger than the typical charge collection times from the 2D-PSD so that the voltage output is approximately independent of the exponential term in equation 3.1.

The effects of the integrating charge sensitive preamplifiers can be simulated in this analysis by replacing the integral term in equation 3.1 by a summation over the time step, p, and neglecting the small exponential capacitive discharge term, such that,

$$V_{a;(m',n')} = \frac{1}{C_{PA}}\sum_{t=0}^{t_d} i_{a;(m',n')} = \frac{4M^2}{C_{PA}CR_T}\sum_{t=0}^{t_d} q_{0,0}$$
$$V_{b;(m',n')} = \frac{1}{C_{PA}}\sum_{t=0}^{t_d} i_{b;(m',n')} = \frac{4M^2}{C_{PA}CR_T}\sum_{t=0}^{t_d} q_{M-1,0}$$
$$V_{c;(m',n')} = \frac{1}{C_{PA}}\sum_{t=0}^{t_d} i_{c;(m',n')} = \frac{4M^2}{C_{PA}CR_T}\sum_{t=0}^{t_d} q_{0,M-1}$$
$$V_{d;(m',n')} = \frac{1}{C_{PA}}\sum_{t=0}^{t_d} i_{d;(m',n')} = \frac{4M^2}{C_{PA}CR_T}\sum_{t=0}^{t_d} q_{M-1,M-1} \tag{3.2}$$

where t_d is the signal development time. The energy signal, V_E, is equal to the sum of the four position signals,

$$V_{E;(m',n')} = V_{a;(m',n')} + V_{b;(m',n')} + V_{c;(m',n')} + V_{d;(m',n')} \tag{3.3}$$

In figures 9*(a)* and *(b)* the quantities $V_{a;(m',n')}$ and $V_{E;(m',n')}$ are plotted as a function of t_d for several values of nodal hit co-ordinate (shown in parentheses) for *mesh-II*, again with $R_T = R_\square/10$. The value of C_{PA} has been set to unity. The same quantities are plotted for *mesh-I* in figures 9*(c)* and *(d)*; note the different time-axis scalings.

The termination resistors act to compress the position signal range from $0 \rightarrow 100\%$ to $\approx 10 \rightarrow 60\%$ of the energy signal. This ensures that the smallest position signals are non-zero. For all four position signals to be detected in the real 2D-PSD the low energy detection threshold must be less than these smallest signals, thus the requirement,

$$\text{Detection threshold} < V_{a;(M-1,M-1)} \times E_{particle} \approx 10\% E_{particle} \tag{3.4}$$

must be fulfilled for all signals to be recorded, where $E_{particle}$ is the minimum incident particle energy to be detected.

In figure 9*(a)*, for $t < 0.05R_\square C$, $V_{a;(10,10)}$ develops quicker than expected, with more charge being delivered to electrode 'a' from a hit at $(10,10)$ than for the closer hit at $(5,5)$. This unexpected behaviour suggests that the charge originating at the furthest point from 'a' has found a quicker route to the electrode than that originating at the centre node. Contour plots of nodal charge density provide a more detailed view of the progress of the charge across the resistive sheet and are shown for *mesh-II* in figures 10*(a)* to *(c)* for the initial condition $q_{10,10}^{t=0} = 1$ for three values of t; *(a)* $t = 5 \times 10^{-4} R_{\Box}C$; *(b)* $t = 2 \times 10^{-3} R_{\Box}C$ and *(c)* $t = 0.01 R_{\Box}C$. The contours are equally spaced, with the greatest charge existing near node $(10, 10)$ in *(a)&(b)* and node $(9, 9)$ in *(c)*. These plots highlight three distinct components to the charge flow, as indicated by the arrows. Two fast components of the charge generated at node $(10, 10)$ flow around the mesh perimeter via nodes $(0, 10)$ and $(10, 0)$ to electrode 'a' at node $(0, 0)$ and are followed by a much slower component which diffuses directly across the resistive surface. This multi-component charge flow is not seen in the uniform resistance *mesh-I* . The fast perimeter charge flow is a direct consequence of the presence of the low resistance ring and also the electrode termination resistors, without which no charge would flow to electrode 'a'. For charge originating near the mesh centre there are no fast perimeter components to the charge flow and hence the charge collection at all electrodes is slower. This has important consequences when considering position reconstruction.

Since the effects of a shaping amplifier are difficult to simulate in this type of numerical analysis no consideration has been given to an integration - differentiation filter; instead, the finite pulse integration time has been simulated by limiting the signal development time, t_d, in equations 3.2. The time to peak pulse height in a single integration - differentiation filter is approximately $\tau_f = R_f C_f$, where R_f and C_f are the filter components. Therefore in this analysis the signal development time has been set equal to τ_f.

B. Position pattern reconstruction

The position of incidence, or nodal hit co-ordinate, can now be measured using the amplitudes $V_{a \to d;(m',n')}$ for specific values of t_d. The algorithms used to determine the position of the charge origin, detailed in equations 1.2 and 1.3 were modified by replacing the values of $I_{a \to d}$ with $V_{a \to d;(m',n')}$, and used to generate position patterns, for sequential hits at all nodes, which are shown in figures 11*(a) mesh-I* and *(b) mesh-II* . The signal development time was limited to $t_d = 0.3 R_{\Box}C$. A frame showing the "true" position of incidence is also plotted for comparison.

The "pin-cushion" distortion evident in figure 11*(a)* (a non-linearity of 5.3% r.m.s) and seen in practical detectors of this type [8], is greatly reduced by the addition of the low resistance ring *(b)* (a non-linearity of 1.2% r.m.s). Maximum distortion occurs at the centre of each edge and is mainly due to the deficit in the signal from the two nearest electrodes. Near each corner electrode the distortion reduces since the largest signal is prompt - resulting in a larger fraction of the charge being recorded - and the pattern approaches its "true" position. The addition of the low resistance ring gives most assistance to charge collection in the areas of the mesh closest to the centre of each edge where the distortion is greatest, whilst having relatively little affect on the charge collection at the closer electrodes - thus linearizing the pattern.

The compression of the position signal range due to the termination resistors limits the range of X and Y. In figure 12*(a)* the position pattern is plotted for *mesh-II* with $R_T = R_\square/200$; the values of X and Y almost reach their maximum range of -1.0 to +1.0, whilst with $R_T = R_\square/10$, figure 12*(b)*, X and Y are restricted to lie between -0.5 and +0.5. However, the relative distortion in the pattern remains unchanged, and these compression effects can be removed for experimental 2D-PSD data during software calibration.

Reducing the value of the signal development time, t_d, causes marked changes in the pattern distortion as the signal deficit increases. A comparison of the non-linearity (% r.m.s.), for both mesh types, for a variety of signal development times is shown in figure 13. In both plots a minimum of distortion occurs at $\approx 0.2 R_\square C$ with a sharp increase in distortion below this value. Also shown are four generated position patterns for *mesh-II* to illustrate the characteristic barrel distortion which appears as the signals are under-shaped. The distortion increases steadily for $t > 0.2 R_\square C$ due to the inexact nature of the reconstruction algorithms when $R_L \neq 0$.

C. Ballistic deficit

The energy signal is much more sensitive to the signal deficit then the position pattern. As is apparent from figures 9*(b)* and *(d)*, the output, $V_{E;(m',n')}$, for a 'mono-energetic' input is a function of incident position, and asymptotically approaches the true value as t_d increases. The maximum deficit occurs in both cases for $(m', n') = (5, 5)$, ie. the mesh centre. The ballistic deficit is defined here as,

$$B_{D;(m'n')} = \left(1 - V_{E;(m'n')}\right) \times 100\% \tag{3.5}$$

The ballistic deficit profile at $t_d = 0.2 R_\square C$ for *mesh-II* is shown in figure 14*(a)*. The deficit is symmetric about the mesh centre and approaches zero near the four corners.

A plot of $log_{10}(B_{D;(5,5)})$ vs. t_d for various values of $R_\square / L R_L$, shown in figure 14*(b)*, reveals a linear relationship between the deficit and the signal development time. Also, as the ratio $R_\square / L R_L$ is increased, the deficit profile approaches a minimum for all t_d. It has been found that B_D is independent of the magnitude of the initial condition, in real terms the deficit is energy independent, facilitating software correction of experimental data; and that the energy signal should be allowed to develop for longer than $2R_\square C/5$ seconds for less than 1% ballistic deficit at the centre of *mesh-II* .

Although the ballistic deficit shown in figure 14*(b)* for the uniform mesh $(R_\square / L R_L = 1)$ is large ($\approx 30\%$), even for large values of t_d ($0.4 R_\square C$), the value of the characteristic 2D-PSD time constant, $R_\square C$, shortens (if R_L is kept constant) as the ratio $R_\square / L R_L$ is decreased towards the uniform mesh resistance limit. This means that a relatively short shaping time, $\tau_f \approx 1.5\mu$s, corresponds to large values of signal development time, $t_d \gg 0.5 R_\square C$, thus reducing the ballistic deficit in practical detectors of this type.

IV. COMPARISON WITH EXPERIMENTAL MEASUREMENTS

The details of the construction of the 2D-PSD's and of the experimental measurements taken with them during the commissioning of MEGHA are reported elsewhere [1]. To assess the validity of the model calculations they have been compared to these experimental data. For MEGHA two types of 2D-PSD have been constructed, one 500μm thick and the other 385μm thick. The model parameters used in this comparison were set to match those of the two 2D-PSD's.

The shape of the preamplifier output voltage pulses from the 385μm thick 2D-PSD were investigated by illuminating the detector with 5.5MeV α-particles. The α-particles were collimated through a $1mm$ diameter hole placed $1mm$ above the 2D-PSD surface to constrain the position of incidence. Four $1k\Omega$ termination resistors were placed in series between the position electrodes and the charge-sensitive preamplifiers and a reverse bias of $100V$ was used to deplete the junction and minimize the detector capacitance. The rear surface energy signal charge pulses were integrated using standard preamplifiers and then recorded using a fast digital oscilloscope for two positions of incidence on the 2D-PSD surface.

Figure 15 shows the typical voltage pulse profiles obtained for an α-particle striking the 2D-PSD corner *(a)* and the 2D-PSD centre *(b)*. Also shown in the figure as dashed lines are the equivalent model profiles for the nodal hit co-ordinates $(0,0)$ (mesh corner) *(a)* and $(5,5)$ (mesh centre) *(b)* for a mesh with $M = 11$. The x and y-axis scales are those from the model; the experimental data has been scaled along the y-axis only so that the maximum output of the 2D-PSD-preamplifier combination corresponds to $V_E = 1.0$.

The model and experimental profiles are in very good agreement. The oscillatory overshoot in figure 15*(a)* is due to the fast ($\approx 100ns$) pulse rise-time nearing the maximum oscilloscope slew rate. This agreement validates the assumptions made in the model and demonstrates that a mesh size of $M = 11$ should give a reasonable representation of the experimentally derived position patterns.

To investigate the position linearity of the detectors measurements [1] were taken with a mask of 17×17 holes of $1mm$ diameter at a separation of $3mm$. The mask was placed $65mm$ in front of the 500μm 2D-PSD, which was placed $600mm$ from a 150μg/cm^2 ^{197}Au target, with the detector centre $\approx 6^o$ from the beam axis. A beam of 85MeV ^{12}C ions were scattered from the target onto the detector. The details of the electronics and data acquisition used for these measurements are given elsewhere [12]. The position signal shaping times used were $\tau_f = 1.5\mu$s$=0.5R_{\Box}C$. Termination resistors of $1k\Omega$ were used and a bias of $100V$ applied to deplete the junction.

The position pattern obtained, showing the image of the mask of holes, is plotted in figure 16*(a)* and shows slight pin-cushion distortion. The equivalent model calculation is overlayed for comparison. Again there is good agreement between the model and experimental data.

The pin-cushion distortion can be corrected by constructing two bi-cubic spline surface fits, X' and Y', of the raw X, Y data to a linear grid representing the true position of incidence. NAG [13] library routines E02DDF and E02DFF were used to construct the spline surfaces. The X' and Y' surfaces vary as a function of X and Y,

$$\begin{aligned} X' &= f(X,Y) \qquad \text{and,} \\ Y' &= g(X,Y) \end{aligned} \tag{4.1}$$

The true position of incidence can be derived from the raw data by sampling the two surfaces at the point (X, Y).

The data points used to construct the spline surfaces can be determined by either (i) using the centroids of the peaks corresponding to the mask holes in figure 16*(a)*, or (ii) using the values calculated by the model. Method (i) offers the advantage that additional non-linearities caused by inhomogeneities in the surface resistances and other effects not incorporated in the model can be removed; however, this process is time consuming, especially for a large array of 2D-PSD's, and also requires additional experimental run-time for position calibration with a mask. Method (ii) is much more general and can be used to correct several 2D-PSD's simultaneously, with a variation of only three parameters needed to fit any combination of surface and ring resistance and shaping time. Method (ii) has been used to improve the non-linearity of the pattern in figure 16*(a)*, the resultant pattern is plotted below in figure 16*(b)* along with the linear grid used to generate the spline surfaces.

During use the characteristics of the 2D-PSD can change due, for example, to radiation damage and increased leakage current. This can affect the position reconstruction as illustrated in figure 17. Figure 17*(a)* shows a position spectrum taken for a 385μm 2D-PSD with 3pnA 170MeV ^{24}Mg ions scattered from a 75μg/cm^2 ^{12}C target. For these measurements the mask of holes had been removed, but a solid circular disc was used to shield the 2D-PSD from ions scattered to angles less than 4.2^o. The cut out in the bottom of figure 17*(a)* shows the shadow cast by this disc. Figure 17*(b)* shows a similar spectrum taken later in the experiment, when the leakage current had risen from 0.6μA to 4.4μA. It can be seen that the deterioration in the position reconstruction has almost removed any evidence of the shadow cast by the disc. Correlated with this increased leakage current was an 8% drop in the ^{24}Mg elastics pulse height, measured from a sum of the four position signals. It is apparent that, as the leakage current increased, more of the applied bias voltage is dropping across an 11$M\Omega$ load resistor placed in series between the bias supply and 2D-PSD, and less across the junction itself, hence the junction was no longer fully depleted. This in turn led to an increase in junction capacitance and a lengthening of the characteristic detector time constant, $R_\square C$. The model would predict that this should result in greater position distortion and ballistic deficit, as shown figures 13 and 14.

These effects can be incorporated into the model by using a reduced signal development time t'_d,

$$t'_d = t_d \frac{C_i}{C_f} \tag{4.2}$$

where C_i and C_f are the initial and final values of junction capacitance respectively. Using values of C_i and C_f calculated from the capacitance-voltage characteristic curve supplied by the manufacturers with the 2D-PSD a model prediction of the position distortion was made. This is superimposed over the experimental data in figure 17*(b)*, and can be seen to give a good representation of the experimental spectrum.

The model calculation can be used to correct for the position distortion using the spline fits described above. The corrected data is plotted in figure 17*(c)* along with the linear grid used to generate the spline surfaces. The fracturing of the pattern towards the edges is caused by the spectrum plotting routine, and is not a real effect. The degree of success in restoring linearity to the pattern can be judged in terms of the shape of the shadow cast by the protective shield - which has almost regained its original semi-circular shape.

Occasionally, during experiments, one or two of the five signals from a 2D-PSD can be lost for various reasons, for example electrode radiation damage or preamplifier/amplifier failure; this normally necessitates replacement of the damaged element. However, using data derived from the present model, in a large number of cases the position information from the 2D-PSD can be obtained from two adjacent position signals.

To illustrate this figure 18*(a)* shows a position spectrum, taken through the mask of $1mm$ holes, using all four position signals. To simulate a damaged 2D-PSD, with only two adjacent position electrodes working, the reduced position spectrum is calculated using electrodes 'a' and 'b' normalized to the energy signal 'E'. This spectrum is plotted in figure 18*(b)* and is overlayed with the equivalent model calculations.

The spline surface fitting method has been used to linearize the pattern; the resultant position spectrum is shown in figure 18*(c)*. Although the corrected pattern possess poorer linearity and resolution than that of figure 18*(a)*, especially towards the top edge furthest from the electrodes 'a' and 'b', this method does allow data from damaged detectors to be recovered.

V. SUMMARY

A new numerical model has been developed that allows accurate simulation of the electrode currents from large area sheet resistance position sensitive detectors both with and without a low resistance ring.

The model allows prediction of the position non-linearity and ballistic deficit from the known sheet resistance, ring resistance and capacitance of the detector; this can be used to optimize the detector design prior to construction.

The model data can also be used to correct for distortions that occur during detector use and to recover data from detector events where all the signals are not recorded.

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FIGURES

FIG. 1. Schematic of the original resistive anode suggested by Gear. The detector comprises a central sheet of resistance $R_\square$ per unit square; the electrodes are linked via a low resistance ring of resistance R_L per unit length.

FIG. 2. Schematic showing part of the resistive anode to illustrate the model formulation. The mesh of resistors, r_s and r_l, join the centres of the M^2 anode segments. Each segment has a capacitance, c_s, to the common backplane.

FIG. 3. *(a)* Schematic of the nodal arrangement for a general internal segment. The currents flow through the resistors joining adjacent segments, and from the capacitor joining the segment to the common backplane, due to the potential differences existing across the nodal mesh.
(b) Truncated nodal picture for an edge node, and *(c)* a corner node. The capacitance of the material surrounding the edge and corner nodes is reduced compared to an internal node due to the reduced area of the segment.

FIG. 4. Charge profiles for a mesh with $R_\square/LR_L = 1$ at times, *(a)* $t = 0$; *(b)* $t = 0.01R_\square C$ and *(c)* $t = R_\square C$. Note the charge-axis scaling is different for each plot.

FIG. 5. Charge profiles for three mesh types, *(a)* $R_\square/LR_L = 1$ and $R_T = 0$; *(b)* $R_\square/LR_L = 10$ and $R_T = 0$; *(c)* $R_\square/LR_L = 10$ and $R_T = R_\square/10$, at time $t = 0.02R_\square C$.

FIG. 6. Schematic showing the currents $i_{a\to d}$ flowing off the 2D-PSD through the termination resistors to the virtual earths of the charge-sensitive preamplifiers (only one of the four is shown). The electrode labeling will be referred to throughout the text.

FIG. 7. *(a)* As the mesh size is increased the value of the mesh variables, for example $q_{0,0}$, tend asymptotically to a constant.
(b) The calculation time increases approximately proportional to M^4 as the mesh size increases.

FIG. 8. Plot showing the temporal development of the four electrode currents for *mesh-II* for the nodal hit co-ordinate $(2, 3)$.

FIG. 9. Plots showing the temporal development of *(a)* $V_{a;m',n'}$ and *(b)* $V_{E;m',n'}$ for *mesh-II* and *(c)* $V_{a;m',n'}$ and *(d)* $V_{E;m',n'}$ for *mesh-I* for the nodal hit co-ordinates $(m', n') = (0, 0), (3, 3), (5, 5)$ and $(10, 10)$. The slowest signal rise-time occurring at the mesh centre $(m', n') = (5, 5)$. The energy signal profiles are symmetric about the mesh centre. Note the different time-axis scaling for plots *(a)&(b)* compared to *(c)&(d)*.

FIG. 10. Contour plots of nodal charge density for *mesh-II* with $R_T = R_\square/10$ at times, *(a)* $t = 5 \times 10^{-4} R_\square C$; *(b)* $t = 2 \times 10^{-3} R_\square C$ and *(c)* $t = 0.01 R_\square C$. The contours are equally spaced, with the greatest charge existing near node $(10, 10)$ in *(a)&(b)* and node $(9, 9)$ in *(c)*. The arrows are drawn to indicate the direction and extent of the flow of the different current components.

FIG. 11. Calculated position patterns for *(a)* *mesh-I* and *(b)* *mesh-II* at time $t_d = 0.3 R_\square C$.

FIG. 12. Plot showing the variation in position pattern compression due to the termination resistors, *(a)* $R_T = R_\square/200$ and *(b)* $R_T = R_\square/10$, for *mesh-II*.

FIG. 13. Comparison of position pattern non-linearity as a function of signal development time for the two mesh types. Four position patterns for *mesh-II* are plotted to show the increasing barrel distortion at low signal development times.

FIG. 14. *(a)* Energy signal ballistic deficit as a function of nodal position for *mesh-II* at time $t_d = 0.2 R_\square C$.

(b) Variation in the ballistic deficit $\{log_{10}(B_{D;(5,5)})\}$ as a function of $R_\square / L R_L$ and t_d.

FIG. 15. Pre-amplifier output voltage pulses and equivalent model profiles (as dashed lines) for the 385μm 2D-PSD when a 5.5MeV α-particle strikes *(a)* a 2D-PSD corner and *(b)* the 2D-PSD centre.

FIG. 16. *(a)* Plot showing the pin-cushion distortion apparent in the position pattern of the 500μm 2D-PSD when illuminated by 85MeV ^{12}C ions through a mask of $1mm$ diameter holes spaced $3mm$ apart. The equivalent model data used to generate the spline correction surfaces is plotted for comparison.
(b) The data from *(a)* is plotted after correction by the spline surface method along with the linear grid used in the mapping.

FIG. 17. *(a)* Original position pattern generated from the 385μm 2D-PSD at the start of the experiment.
(b) The distorted pattern from the same 2D-PSD later in the experiment. The equivalent model data used to generate the spline correction surfaces is plotted for comparison.
(c) The data from *(a)* is plotted after correction by the spline surface method along with the linear grid used in the mapping.

FIG. 18. *(a)* Position pattern from the 385μm 2D-PSD generated using all four position signals.
(b) The reduced position pattern generated using the two position signals V_a and V_b normalised to the energy signal V_E. The equivalent model data used to generate the spline correction surfaces is plotted for comparison.
(c) The data from *(b)* is plotted after linearisation by the spline surface method.