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A Model of One Proton Emission from Deformed Nuclei

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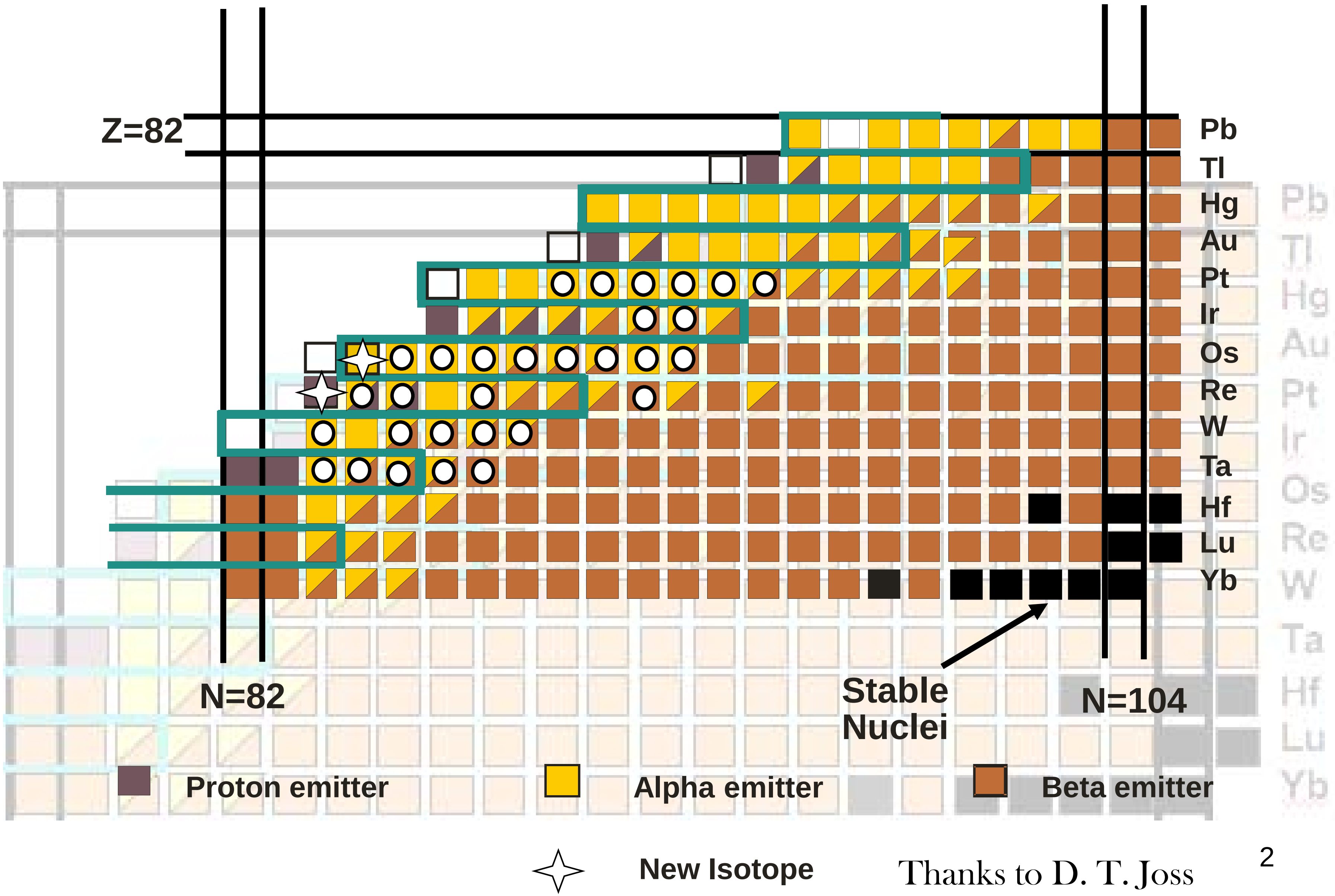
The Z≤82 Region



Thanks to D. T. Joss

- Many nuclei at or beyond proton dripline are unstable against proton emission.
- Initially, parent nucleus is in a *quasi-stationary state*, and proton decay may be considered as a process where the proton tunnels through a potential barrier.
- Large Coulomb potential and centrifugal barrier mean this can often extend out to almost *100* fm.
- Consequently, measurement of decay half-lives (ranging from microseconds to seconds) provides reliable spectroscopic information.
- In principle, proton decay should be easy to model (since no preformation factors like alpha particles).
- Reviews:

  Experimental: Woods and Davids, Ann. Rev. Nucl. Part. Sci. **47** (1997) 541

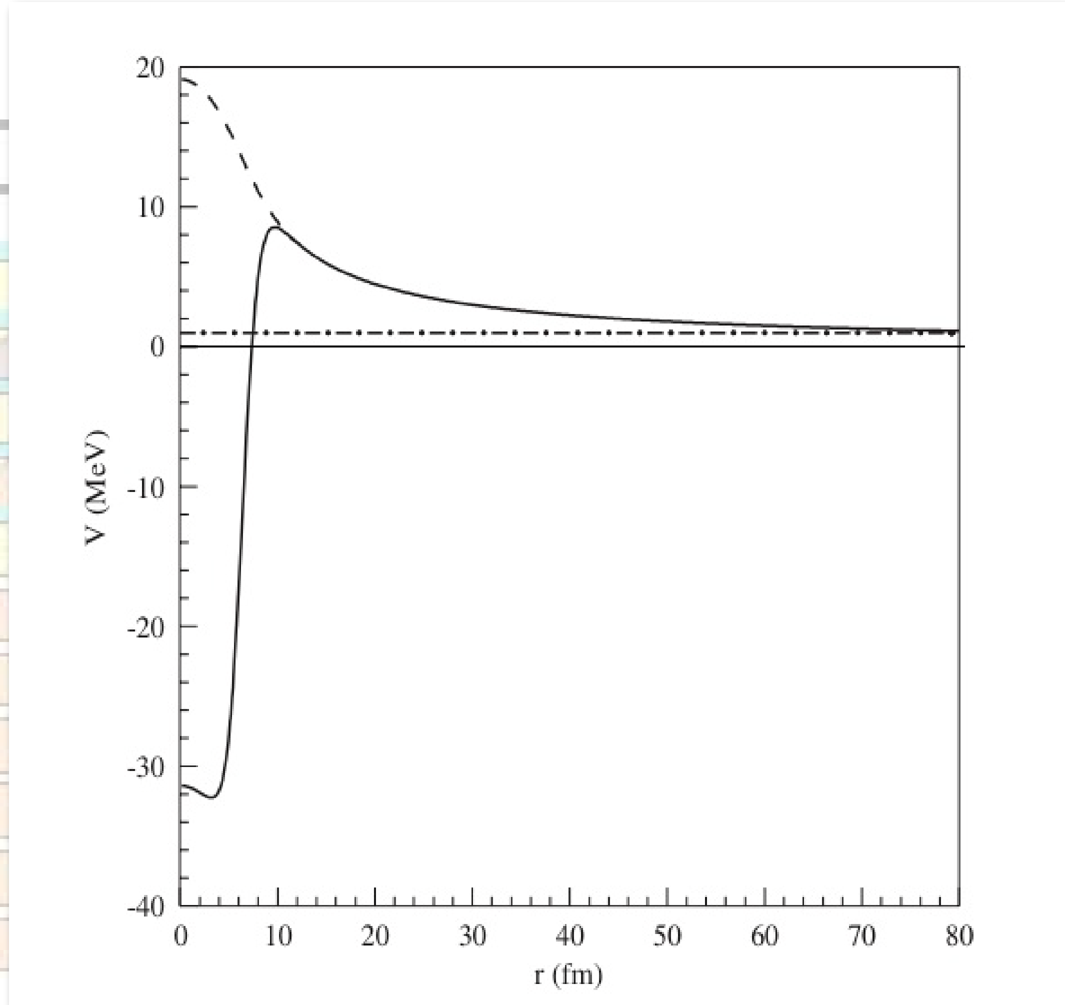
  Theoretical: Delion, Liotta and Wyss, Phys. Rep. **424** (2006) 113.

Spherical Nuclei

Proton decay involves tunnelling through the potential barrier

Total potential consists of nuclear + coulomb + centrifugal parts.

The nuclear part is typically a *mean-field* potential and one has many choices of potential to take



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WKB

WKB is a semi-classical approximation. Inside nucleus, particle moves in classical-like orbit. Each time the barrier is hit, there is a probability that penetration will occur. The width and half life are given by

$$\Gamma = f \exp\left(-\frac{2}{\hbar}\int_{r_1}^{r_2} \sqrt{2\mu\left[V(r) - E\right]}\,dr \right) \qquad t_{1/2} = \frac{\hbar \log 2}{\Gamma}$$

where ***f*** is *knocking frequency* (with which the proton hits against the potential wall).

The potential $V(\mathbf{r})$ is felt by the proton and calculated by various means – e.g. Hartree Fock potential. It can also be used to estimate f.

In the limit of constant potential inside the nucleus, the knocking frequency is based on a classical proton velocity and the nuclear radius:

$$f = \frac{v}{2R_0}$$

Or: Z.A. Dupre and T.J. Burvenich, Nucl Phys A 767 (2006)

Quasistationary picture

- Nuclear decays are time-dependent processes.
- However, typical decay lifetimes v. long on nuclear timescale

  => stationary state problem (Gamow 1928).

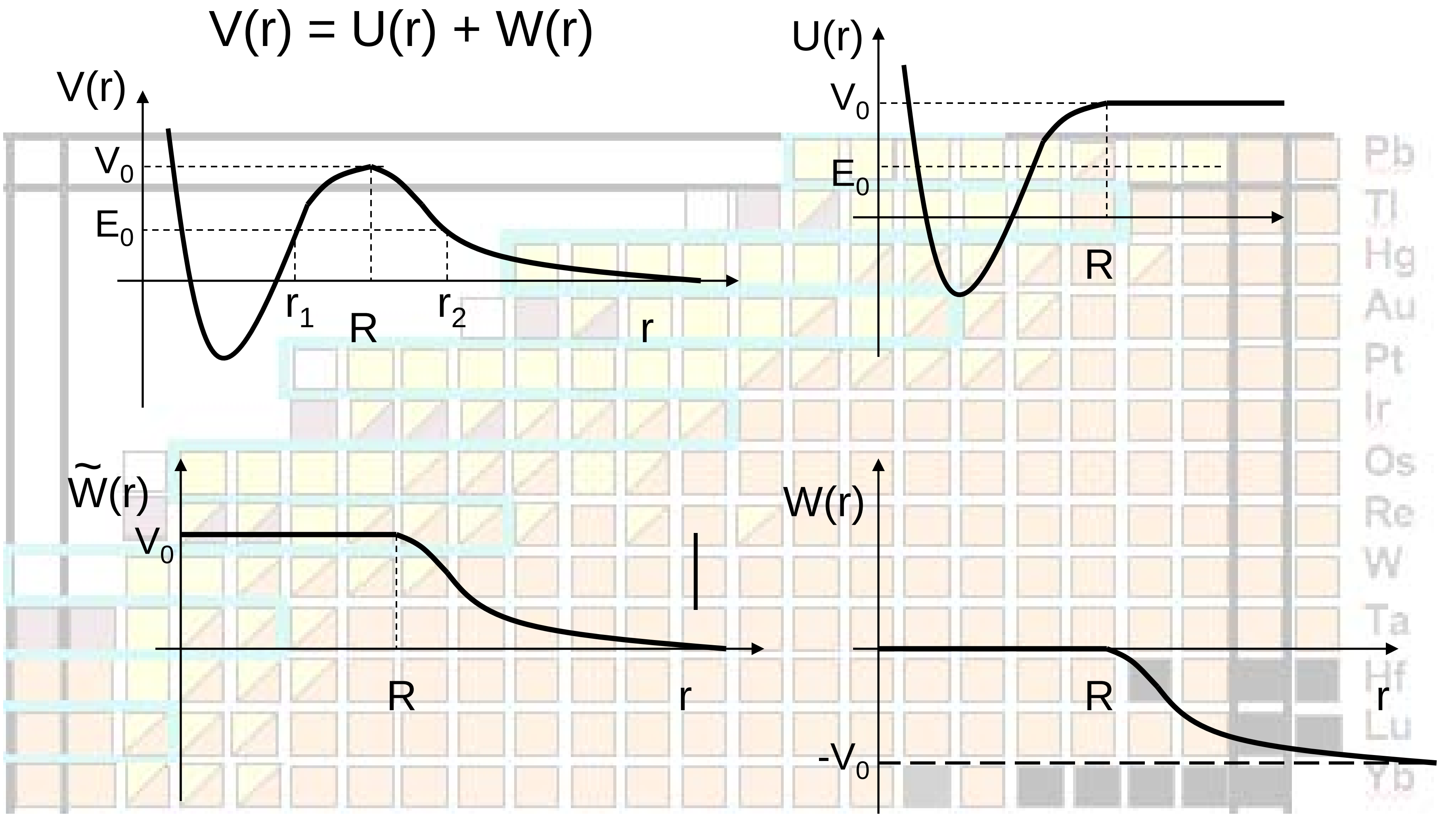
- Usually treated as scattering problem (poles in S-matrix or resonances in scattering amplitude). This is the time reverse of decay, that is wave comes in from infinity and becomes trapped by potential.
- However, quasistationary states of interest here might be closer to bound states (despite having continuous rather than discrete spectrum).
- Can use perturbative method reliably by starting from bound state problem.

Two-Potential approach

Alternative method is known as the Two Potential approach (TPA).

This is a fully quantum method, using perturbation theory developed by Shmuel Gurvitz.

- S.A. Gurvitz and G Kalbermann, Phys. Rev. Lett. 59 (1987) 262
- S. A. Gurvitz, Phys. Rev. A **38** (1987) 1747
- S. Åberg, P. Semmes and W. Nazarewicz, Phys. Rev. C **56** (1997) 1762
- S. A. Gurwitz, P.B. Semmes, W. Nazarewicz and T.Vertse, Phys. Rev. A **69** (2004) 042705


V(r) = U(r) + W(r)
V(r)
V0
E0
r1
R
r2
r
U(r)
V0
E0
R
W̃(r)
V0
R
r
W(r)
R
r
-V0

TPA: Theory

Consider the unperturbed bound state $|\Phi_0\rangle$ in the potential $U(\mathbf{r})$ with eigenvalue E_0 :

$$H_0|\Phi_0\rangle = E_0|\Phi_0\rangle \qquad H_0 = -\frac{h^2}{2\mu}\nabla^2 + U(r)$$

The perturbation, $W(\mathbf{r})$ transforms it to a quasistationary state. When the potential $W(\mathbf{r})$ is switch on at $t=0$, the state $|\Phi_0\rangle$ becomes the wavepacket and we can derive a simple expression for the decay width

$$\Gamma = \frac{4\mu}{h^2 k}\left|\int_R^\infty \varphi_0(r)W(r)\chi_k(r)\,dr\right|^2$$

$$i\frac{\partial}{\partial t}|\Phi_0(t)\rangle = H_0|\Phi_0(t)\rangle$$

Radial bound state wfn in potential *U(r)*

Scattering wfn due to potential $\tilde{W}(r)$

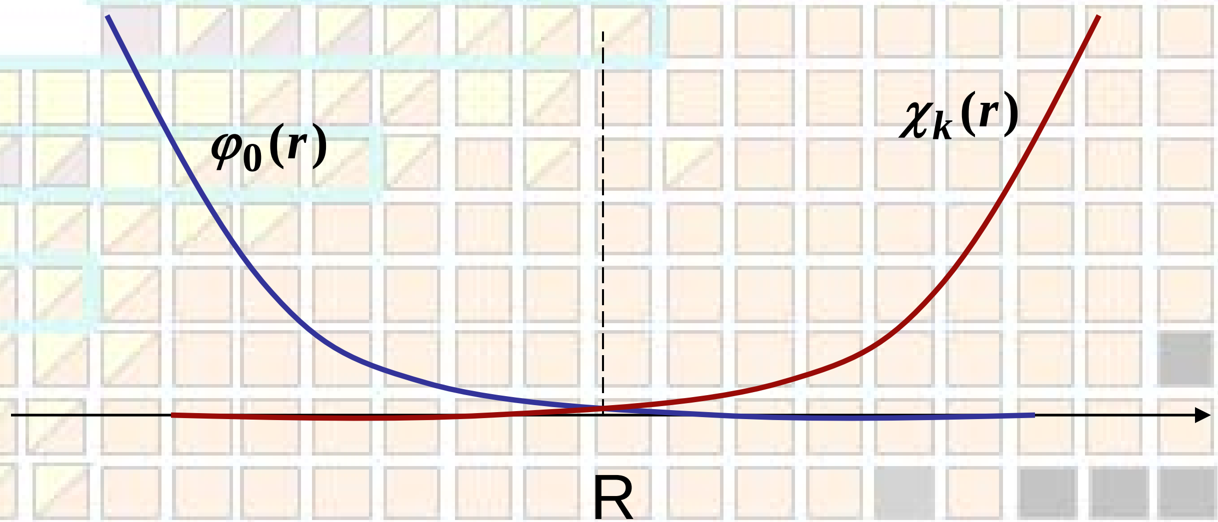
TPA: Theory

Since R is deep inside barrier, both bound and scattering states are simple exponentials in this region:

$$\varphi(r) = \varphi(R)e^{-\alpha(r-R)}$$

$$\chi(r) = \chi(R)e^{+\alpha(r-R)}$$

$$\alpha = \frac{\sqrt{2\mu(V_0 - E_0)}}{\hbar}$$



TPA: Theory

This allows the integral to be solved exactly:

$$\Gamma = \frac{4h^2\alpha^2}{\mu k}\left|\varphi_0(R)\chi_k(R)\right|^2$$

And a final simplification can be made to the scattering wave function outside the influence of the nuclear potential:

$$\chi_k(r) = \cos\delta_l\, F_l(kr) - \sin\delta_l\, G_l(kr)$$

Since the phase shift is so small inside the barrier, the second term above is negligible and to a good approximation

$$\chi_k(r) \approx F_l(kr)$$

Thus,

$$\Gamma = \frac{4h^2\alpha^2}{\mu k}\left|\varphi_0(R)F_l(kR)\right|^2$$

S. Åberg, P. Semmes and W. Nazarewicz, Phys. Rev. C **56** (1997) 1762

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Results

Proton half-lives for various proton emitters calculated within the WKB and TPA models using a Hartree-Fock potential based on the SkP Skyrme force and compared with the measured values.

Nucleus	State	Energy (MeV)	Half-lives (s) WKB	TPA	Exp
^{147}Tm	$0h_{11/2}$	1.071	23.4	17.8	3.9
^{151}Lu	$0h_{11/2}$	1.255	0.270	0.221	0.1271
^{155}Ta	$0h_{11/2}$	1.444$^{(a)}$	7.8×10^{-3}	6.3×10^{-3}	2.9×10^{-3}
^{157}Ta	$2s_{1/2}$	0.947	0.60	0.47	0.30
^{159}Re	$0h_{11/2}$	1.805$^{(b)}$	2.45×10^{-5}	2.04×10^{-5}	2.1×10^{-5}
^{161}Re	$2s_{1/2}$	1.214	3.9×10^{-4}	3.3×10^{-4}	3.7×10^{-4}
^{161}Re	$0h_{11/2}$	1.338	0.19	0.15	0.325
^{165}Ir	$0h_{11/2}$	1.733	1.5×10^{-4}	1.3×10^{-4}	3.5×10^{-4}
^{167}Ir	$2s_{1/2}$	1.086	0.069	0.056	0.11
^{171}Au	$0h_{11/2}$	1.718	3.9×10^{-4}	3.5×10^{-4}	2.22×10^{-3}

Summary for Spherical Emitters

- Proton emission using spherical potentials with WKB or TPA give reasonable results and in close agreement
- In principle, HF mean field potential provides proton energy too.
- In practice, the potential has to be scaled slightly to give correct proton energy.
- Underlying mean field cannot predict single particle energies sufficiently well.

Deformed proton emitters

Currently several models on market to calculate partial decay widths for deformed nuclei:

- Gamow states method in adiabatic limit (complex energies)

  E. Maglione, L.S. Ferreira & R. J. Liotta, Phys Rev Lett **81** (1998) 538

- Coupled Channels method (real energies)

  H. Esbensen & C. N. Davids, Phys Rev C **63** (2000) 014315

Both start from scattering state picture.

e.g. first method:

Partial decay width to proton channel (ℓj) is given by radial probability flux through a sphere of radius *R* outside the nuclear potential

$$\Gamma_{\ell j} = \frac{\hbar^2 k}{m\left(j+1/2\right)} \frac{R^2\left|\psi_{\ell j}(R)\right|^2}{F_\ell^2(R)+G_\ell^2(R)}$$

Proton single particle wave function in deformed potential

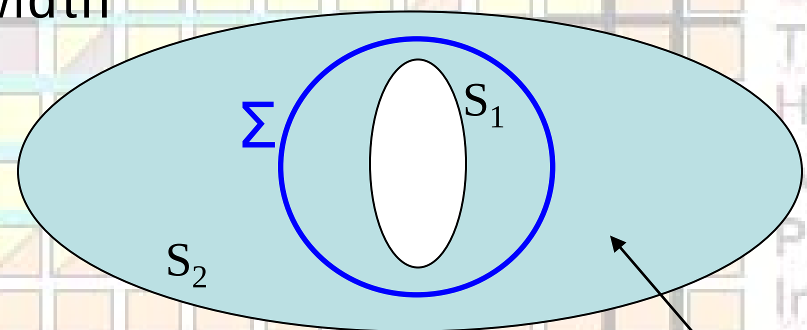
Coulomb functions

TPa for deformed potentials

As an alternative approach, can develop TPA in 3-D:

For a deformed potential, barrier width now has angular dependence

S_1 and S_2 denote equipotential lines for a proton with energy E_0



Shaded area is classically forbidden region

Instead of a cut-off radius we now Have a separation surface, Σ, dividing the potential into inner and outer regions as before:

$$V(\mathbf{r}) = U(\mathbf{r}) + W(\mathbf{r})$$

TPA for deformed potentials

We then proceed as in 1-D case.

[This has been developed by Gurvitz in
Multiple Facets of Quantization and Supersymmetry: Michael Marinov Memorial Volume, Ed. by M Olshanetsky, A. Vainshtein]

We obtain then a proton partial decay width

$$\Gamma_k = \frac{1}{4mk_0}\int_\Sigma \left[\Phi_0(\mathbf{r})\nabla\varphi_{\mathbf{k}}(\mathbf{r}) - \varphi_{\mathbf{k}}(\mathbf{r})\nabla\Phi_0(\mathbf{r}) \right] d\sigma$$

where we have used the divergence theorem to give a surface integral.

This needs to be solved with correct angular momentum projection onto decay channel.

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Summary and Acknowledgements

It is hoped that the 3-D TPA model of Gurvitz can be applied to study the single proton emission from deformed nuclei and compared with existing scattering state methods.

This study formed part of a project with Paul Stevenson at Surrey, Robert Page and Dave Joss at Liverpool and John Simpson at Daresbury.